PERGAMON

International Journal of Solids and Structures 36 (1999) 517–531

INTERNATIONAL JOURNAL OF SOLIDS and STRUCTURES

Dynamic interaction between a matrix crack and a circular inhomogeneity with a distinct interphase

X. D. Wang, S. A. Meguid*

Engineering Mechanics and Design Laboratory, Department of Mechanical and Industrial Engineering, University of Toronto, Toronto, Ontario, Canada M5S 3G8

Received 10 December 1996; in revised form 14 January 1998

Abstract

This article provides a theoretical treatment of the dynamic interaction between a matrix crack and an arbitrarily located circular inhomogeneity with a distinct interphase under antiplane loading. The matrix/inhomogeneity interphase is characterized by a linear spring model. The theoretical formulations governing the steady state problem are based upon the use of integral transform techniques, Bessel function expansions and a Pseudo-incident wave technique. The closed form expression for the resulting stress intensity factor at the matrix crack is obtained by solving the appropriate singular integral equations using Chebyshev polynomials. Typical examples are provided to show the effect of the location of the inhomogeneity, the material combination and the interface property upon the dynamic stress intensity factor of the matrix crack.

1. Introduction

A comprehensive body of knowledge exists dealing with the elastostatic behaviour of interacting reinforced fibres and particles (inhomogeneities) in metal matrix composites, see e.g., the reviews by Mura (1987) and Evans (1990). Most of the quasistatic available solutions assumed that only two materials with a common interface exist in the composite solid. It is often the case, however, that the reinforcing fibres or particles are surrounded by a thin interfacial layer (interphase). This interphase may be created unintentionally as the result of chemical interaction between the constituents, or intentionally to enhance the mechanical properties of the composite solid (McCullough, 1971; Chiu et al., 1994; Wang and Chiang, 1996). The existence of and the interaction between these coated fibres ultimately govern the mechanical behaviour and the overall failure mechanism of this class of materials (Achenbach and Zhu, 1990; Chen et al., 1990; Wass, 1992; Drzal and Madhukar, 1993). Accurate assessment of the fracture response of these materials would thus require a reliable assessment of the possible interaction effects between these fibres.

* Corresponding author. Fax: 4169785741.

0020–7683/99/$ - see front matter
PII: S0020–7683(98)00039–0

The corresponding elastodynamic problem has recently evoked interest among researchers working in the field of nondestructive characterization of interfaces and materials engineering. The work was mostly motivated by the desire to establish the mechanical properties of interface (Datta et al., 1988, Olssen et al., 1990; Karpur et al., 1994), the scattering behaviour of the wave (Schafbuch et al., 1990) and material response to dynamic loading (Ramesh and Ravichandran, 1990; Meguid and Wang, 1994).

The objective of the present paper is to provide an analytical treatment of the dynamic interaction between a matrix crack and an arbitrarily located inhomogeneity with distinct interphase subject to different antiplane incident waves. The interfacial layer between the matrix and the inhomogeneity is modelled in terms of linear springs. The analysis is based upon the use of a Pseudo-incident wave technique together with the application of Fourier integral transforms for the crack and Bessel function expansions for the inhomogeneity. Two aspects of the work are accordingly examined. The first is concerned with determining the effect of the interfacial property, material mismatch and loading frequency upon the resulting dynamic stress intensity factor of the matrix crack, while the second is associated with the possible shielding and amplification effects observed in the equivalent static problem.

2. Formulation of the problem

2.1. *Description of problem*

The situation envisaged is that of an elastic infinitely extended isotropic solid containing a matrix crack of length $2a$ and an arbitrarily located circular inhomogeneity of radius R, as shown in Fig. 1. The matrix and the inhomogeneity are connected by a thin layer of thickness h. The shear moduli of the matrix, the inhomogeneity and the interphase are assumed to be μ_M, μ_F and μ_{in}, and the corresponding shear wave speeds c_M, c_F and c_{in}, respectively. A Cartesian (x, y) and a polar

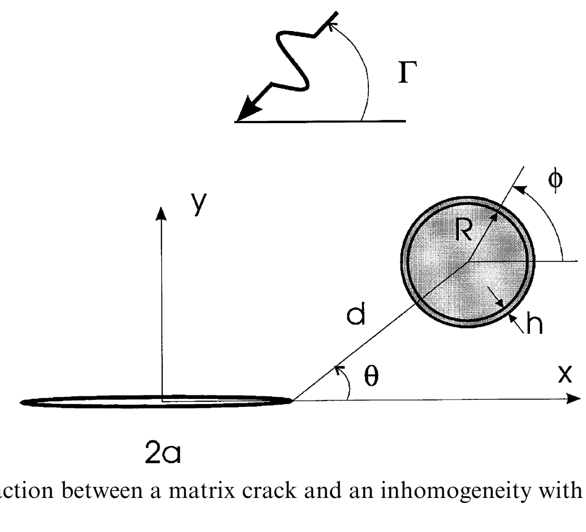

Fig. 1. Interaction between a matrix crack and an inhomogeneity with an interphase.

(r, ϕ) coordinate systems are used to characterize the crack and the inhomogeneity. The distance between the right tip of the crack and the centre of the inhomogeneity is denoted d and the inclination angle of the inhomogeneity centre with respect to the x-axis is denoted θ.

The solid is subjected to a steady state antiplane wave directed at an angle Γ with the x-axis, which can be expressed as

$$\begin{aligned}
\bar{\tau}^0_{xz}(x, y, t) &= \tau^0_{xz}(x, y)\, e^{-i\omega t} = \tau \cos\Gamma\, e^{-ik_M(x\cos\Gamma + y\sin\Gamma)}\, e^{-i\omega t} \\
\bar{\tau}^0_{yz}(x, y, t) &= \tau^0_{yz}(x, y)\, e^{-i\omega t} = \tau \sin\Gamma\, e^{-ik_M(x\cos\Gamma + y\sin\Gamma)}\, e^{-i\omega t} \\
\bar{w}^0(x, y, t) &= w^0(x, y)\, e^{-i\omega t} = \frac{i\tau}{k_M\mu_M} e^{-ik_M(x\cos\Gamma + y\sin\Gamma)}\, e^{-i\omega t}
\end{aligned} \tag{1}$$

where w^0, τ^0_{xz} and τ^0_{yz} are the amplitudes of the displacement and stresses, ω is the circular frequency, and $k_M = \omega/c_M$ is the wave number with c_M being the shear wave speed of the matrix.

The displacement field of the current medium under such an incident wave will satisfy the following Helmholtz equations (Achenbach, 1973),

$$(\nabla^2 + k_M^2)w = 0 \quad \text{in the matrix} \tag{2}$$

$$(\nabla^2 + k_F^2)w = 0 \quad \text{in the homogeneity} \tag{3}$$

in which ∇^2 is the Laplacian operator, $k_F = \omega/c_F$ is the wave numbers with c_F being the shear wave speed of the inhomogeneity. For the sake of convenience, the time factor $\exp(-i\omega t)$ which applies to all the field parameters has been suppressed. The non-vanishing stress components in the matrix and the inhomogeneity, in a Cartesian system (x, y), are

$$\tau_{xz} = \gamma_{xz} \begin{cases} \mu_M & \text{in the matrix} \\ \mu_F & \text{in the inhomogeneity} \end{cases} \tag{4}$$

$$\tau_{yz} = \gamma_{yz} \begin{cases} \mu_M & \text{in the matrix} \\ \mu_F & \text{in the inhomogeneity} \end{cases} \tag{5}$$

with γ_{xz} and γ_{yz} being the strain components. The corresponding stresses in the interphase can be expressed in a polar system (r, θ) as

$$\tau_{rz} = \mu_{in}\gamma_{rz}, \quad \tau_{\theta z} = \mu_{in}\gamma_{\theta z} \tag{6}$$

2.2. *Interface model*

For fibre reinforced composites, the interphase between the matrix and the fibre plays a dominant role in characterizing the mechanical behaviour of composite (Matikas and Karpur, 1993; Veazie and Qu, 1995). In fact, the interphase determines the ability of the composite to transfer load between the matrix and the fibre, and as a result governs its toughness. Although complex interface waves may exist at the interphase (Jones and Whittier, 1967), the fact that the thickness of that interphase is usually very small leads to the development of simplified interphase models. For examples, Hashin (1990) studied the effect of an imperfect interface upon the equivalent thermoelastic properties of fibre reinforced materials by introducing a linear relation between the tractions and displacement jumps at the interface. The corresponding dynamic interface model has been used in the non-destructive characterization of the interfacial properties of fibre reinforced composites using ultrasonic waves (Matikas and Karpur, 1993; Karpur et al., 1994, 1995). In the present study, it is assumed that the thickness of the interfacial layer is much smaller than the radius of the fibre and the wave length. This implies that the inertia effects of the interfacial layer can be ignored. According to this model, the radial shear stress τ_{rz} and strain γ_{rz} are assumed to be uniform across the thickness of the interphase, i.e.

$$\tau_{rz}|_{\text{interphase}} = \tau_{rz}(\theta), \quad \gamma_{rz}|_{\text{interphase}} = \gamma_{rz}(\theta) \tag{7}$$

This assumption indicates that

$$\gamma_{rz}(\theta) = \frac{1}{h}[w(R+h,\theta)|_{\text{matrix}} - w(R,\theta)|_{\text{inhomogeneity}}] \tag{8}$$

Therefore, substituting (7) and (8) into (6) yields

$$\gamma_{rz}(\theta) = \beta[w(R+h,\theta)|_{\text{matrix}} - w(R,\theta)|_{\text{inhomogeneity}}] \tag{9}$$

where

$$\beta = \frac{\mu_{in}}{h} \tag{10}$$

is the interfacial stiffness. If h in eqn (9) is assumed to be zero and β is regarded as the only governing parameter of the interface, this model is the same as that used by Hashin (1990) and Matikas and Karpur (1993). It can be seen that such an assumption reduces the original interfacial layer into an interfacial spring with β being the spring stiffness. When $\beta \to \infty$, eqn (9) leads to

$$w(R+h,\theta)|_{\text{matrix}} = w(R,\theta)|_{\text{inhomogeneity}} \tag{11}$$

which indicates that the matrix and the inhomogeneity are perfectly bonded. When $\beta \to 0$, eqn (9) leads to

$$\tau_{rz}(\theta)|_{\text{interphase}} = 0 \tag{12}$$

which is associated with the traction free condition of a hole.

2.3. *Pseudo-incident wave method*

To avoid the difficulties associated with the complex boundary and interfacial conditions, the original dynamic interaction problem will be decomposed into simpler subproblems which involve either the crack or the inhomogeneity, as shown in Fig. 2. These subproblems will then be summed up to provide the superimposed solution of the original problem.

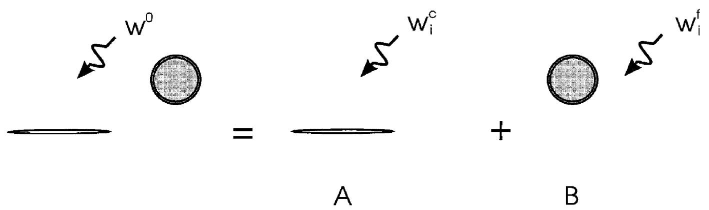

Fig. 2. Subproblems subjected to Pseudo-incident waves.

Let us consider the crack problem first (Fig. 2(A)). The cracked infinite medium is subjected to a Pseudo-incident wave w_i^c which consists of the original incident wave w^0 and the unknown scattering wave from the inhomogeneity, w^f, i.e.

$$w_i^c = w^0 + w^f \tag{13}$$

The corresponding Pseudo-incident stress field can then be expressed as

$$\tau_{jz}(w_i^c) = \tau_{jz}(w^0) + \tau_{jz}(w^f), \quad j = x, y \tag{14}$$

The reflection of this incident wave by the crack surfaces will result in a scattering wave w^c in the matrix. To ensure the traction free condition along the crack surface, the superposition of the incident wave and the scattering wave should give zero shear stress at the crack surface, i.e.

$$\tau_{yz}(w_i^c)|_{\text{crack}} + \tau_{yz}(w^c)|_{\text{crack}} = 0 \tag{15}$$

which provides the boundary condition from which the single crack problem can be solved.

Consider now the single inhomogeneity problem (Fig. 2(B)). The infinite medium is subjected to a Pseudo-incident wave w_i^f which is the superposition of the original incident wave w^0 and the unknown scattering wave from the crack, w^c, i.e.

$$w_i^f = w^0 + w^c \tag{16}$$

The corresponding Pseudo incident stress field can be expressed as

$$\tau_{jz}(w_i^f) = \tau_{jz}(w^0) + \tau_{jz}(w^c), \quad j = x, y \tag{17}$$

As a result of this incident wave, a scattering wave w^f in the matrix and a dynamic field w^F in the inhomogeneity are formed. The interfacial conditions discussed in the previous section indicate the existence of the following relations:

$$\tau_{rz}(R+h, \theta)|_{\text{matrix}} = \tau_{rz}(R, \theta)|_{\text{inhomogeneity}} \tag{18}$$

$$\tau_{rz}(R, \theta)|_{\text{inhomogeneity}} = \beta[w(R+h, \theta)|_{\text{matrix}} - w(R, \theta)|_{\text{inhomogeneity}}] \tag{19}$$

which provide the continuity conditions for the solution of the single inhomogeneity problem. The solution of the original problem can be expressed in terms of a single crack and a single inhomogeneity solution, w^c, w^f and w^F, as being

$$w = \begin{cases} w^0 + w^c + w^f & \text{in the matrix} \\ w^F & \text{in the inhomogeneity} \end{cases} \tag{20}$$

and the corresponding stress field is given by

$$\tau_{jz} = \begin{cases} \tau_{jz}(w^0) + \tau_{jz}(w^c) + \tau_{jz}(w^f) & \text{in the matrix} \\ \tau_{jz}(w^F) & \text{in the inhomogeneity} \end{cases} \quad j = x, y \tag{21}$$

The boundary conditions of the original problem at the crack surface and the continuity conditions along the interface between the inhomogeneity and the matrix are satisfied in the solutions of subproblems A and B. In addition, the radiation condition at infinity can be satisfied by assuming that the scattering fields (w^c and w^f) vanish at points far from the crack and the inhomogeneity.

3. Dynamic interaction problem

3.1. *Solution of subproblems*

The solution of the original interaction problem can be obtained by using Pseudo-incident wave method which considers the behaviour of a single crack and a single inhomogeneity solution. Consider now a single crack subjected to a dynamic antiplane loading. By using Fourier transform, the general solution of the displacement and stress fields can be expressed in terms of the following dislocation density function:

$$\psi(x) = \frac{\partial w^c(x, 0^+)}{\partial x}, \quad |x| \leqslant a \tag{22}$$

with $w^c(x, 0^+)$ being the displacement of the upper surface of the crack. The total stress field can be decomposed into the incident field and the scattering field. To maintain the original traction free condition of the crack surface (Fig. 2(A)), the crack surface boundary (in the scattering problem) is assumed to be subjected to a shear stress due to the Pseudo-incident wave w_i^c but in an opposite direction, i.e. $\tau_{yz}|_{\text{crack}} = -\tau_{yz}(w_i^c)|_{\text{crack}}$ [eqn (15)]. The imposition of this boundary condition results in a singular integral equation for $\psi(x)$ which involves the well-known square-root singularity. This singular integral equation can be solved by using Chebyshev polynomials expansion of $\psi(x)$, as

$$\psi(x) = \sum_{n=0}^{\infty} \frac{A_n}{\sqrt{1 - \frac{x^2}{a^2}}} T_n\left(\frac{x}{a}\right) \tag{23}$$

Integrating (22) and using the orthogonality conditions of the Chebyshev polynomials lead to $A_0 = 0$. By truncating the Chebyshev polynomials to the Nth term and considering the boundary conditions at N-collocation points along the crack surface, the solution for A_n can be expressed as

$$\{A\} = [S_1]\{\tau_i^c\} \tag{24}$$

where $\{A\} = \{A_1, A_2, \ldots, A_N\}^T$, $\{S_1\}$ is a known matrix and

$$\{\tau_i^c\} = \{\tau_1, \tau_2, \ldots, \tau_N\}^T \tag{25}$$

being the boundary stress due to the Pseudo-incident wave at the collocation points of the crack surface (refer to Appendix A for details). According to eqn (14), this boundary stress can be expressed as

$$\{\tau_i^c\} = \{\tau^0\} + \{\tau^f\} \tag{26}$$

with $\{\tau^0\}$ and $\{\tau^f\}$ being the shear stress at the collocation points along the crack surface due to the original incident wave and the scattering wave of the inhomogeneity problem, respectively.

According to this solution, the scattering displacement and stress of the crack problem along the interphase boundary ($r = R+h$) can be described in terms of $\{A\}$ in the following form:

$$w^c(\phi) = w^c(\bar{x}, \bar{y}) = [F_1(\phi)]\{A\} \tag{27}$$

$$\tau^c_{rz}(\phi) = \tau^c_{xz}(\bar{x}, \bar{y})\cos\phi + \tau^c_{yz}(\bar{x}, \bar{y})\sin\phi = [F_2(\phi)]\{A\} \tag{28}$$

where

$$[F_1(\phi)] = \{p_1(\bar{x}, \bar{y}), p_2(\bar{x}, \bar{y}), \ldots, p_N(\bar{x}, \bar{y})\} \tag{29}$$

$$[F_2(\phi)] = \{\bar{p}_1(\bar{x}, \bar{y}), \bar{p}_2(\bar{x}, \bar{y}), \ldots, p_N(\bar{x}, \bar{y})\} \tag{30}$$

and

$$\bar{x} = a + d\cos\theta + (R+h)\cos\phi \quad \text{and} \quad \bar{y} = d\sin\theta + (R+h)\sin\phi \tag{31}$$

with $p_1, p_2, \ldots, p_N$; $\bar{p}_1, \bar{p}_2, \ldots, \bar{p}_N$ being known functions given in Appendix A.

Let us now consider the displacement and stress fields due to a single inhomogeneity with an interphase subjected to a Pseudo-incident wave w^f_i. The general solution of the displacement field of the present problem can be expressed in a polar coordinate system (r, ϕ), as shown in Fig. 1, as

$$w^f(r, \phi) = \begin{cases} \sum_{n=0}^{\infty} H^{(1)}_n(k_M r)[a_n e^{in\phi} + b_n e^{-in\phi}] & \text{in the matrix} \\ \sum_{n=0}^{\infty} J_n(k_F r)[c_n e^{in\phi} + \mathrm{d}_n e^{-in\phi}] & \text{in the homogeneity} \end{cases} \tag{32}$$

where $H^{(1)}_n$ and J_n are Hankel function and Bessel function of the first kind, respectively. The displacement field in the interphase can be obtained by using the interphase model discussed in the previous section as

$$w(r, \phi)|_{\text{interphase}} = w(R, \phi) + \frac{r-R}{h}[w(r+h, \phi) - w(R, \phi)] \tag{33}$$

The solution of a_n, b_n, c_n and d_n corresponding to an incident wave w^f_i be obtained by making use of the interphase model, eqns (18) and (19), such that:

$$\begin{Bmatrix} a_n \\ c_n \end{Bmatrix} = [K_n] \int_0^{2\pi} \begin{Bmatrix} w^f_i|_{\text{inter}} \\ \tau_{rz}(w^f_i)|_{\text{inter}} \end{Bmatrix}_{r=R+h} e^{-in\phi}\,\mathrm{d}\phi \tag{34}$$

$$\begin{Bmatrix} b_n \\ d_n \end{Bmatrix} = [K_n] \int_0^{2\pi} \begin{Bmatrix} w^f_i|_{\text{inter}} \\ \tau_{rz}(w^f_i)|_{\text{inter}} \end{Bmatrix}_{r=R+h} e^{in\phi}\,\mathrm{d}\phi \tag{35}$$

where

$$[K_n] = -\frac{1}{2\pi}\begin{bmatrix} H^{(1)}_n[k_M(R+h)] & -J_n(k_F R) - h\mu_F k_F J'_n(k_F R)/\mu_{in} \\ \mu_M k_M H^{(1)\prime}_n[k_M(R+h)] & -\mu_f k_F J'_n(k_F R) \end{bmatrix}^{-1} \tag{36}$$

with the prime ($'$) representing the derivative with respect to the variable in the parentheses.

3.2. *Interaction between the crack and the inhomogeneity*

The single crack and the single inhomogeneity problems discussed above are coupled in the sense that the scattering wave from the crack subproblem becomes the incident wave for the inhomogeneity subproblem. In addition, the resulting scattering wave from the inhomogeneity subproblem becomes the incident wave for the crack subproblem. These relations will be used to determine the coupled solution of the original problem. Since the Pseudo-incident wave of the inhomogeneity subproblem consists of the original incident wave and the scattering wave of the crack subproblem, by making use of (27) and (28), the solution given by (34) and (35) can be rewritten in terms of the solution of the crack problem, $\{A\}$, as follows

$$\begin{Bmatrix} a_n \\ c_n \end{Bmatrix} = [K_n] \int_0^{2\pi} \left(\begin{bmatrix} F_1(\phi) \\ F_2(\phi) \end{bmatrix} \{A\} + \begin{bmatrix} w^0(\phi) \\ \tau_{rz}^0(\phi) \end{bmatrix} \right) e^{-in\phi} \, \mathrm{d}\phi \tag{37}$$

$$\begin{Bmatrix} b_n \\ d_n \end{Bmatrix} = [K_n] \int_0^{2\pi} \left(\begin{bmatrix} F_1(\phi) \\ F_2(\phi) \end{bmatrix} \{A\} + \begin{bmatrix} w^0(\phi) \\ \tau_{rz}^0(\phi) \end{bmatrix} \right) e^{in\phi} \, \mathrm{d}\phi \tag{38}$$

where $w^0(\phi)$ and $\tau_{rz}^0(\phi)$ are the displacement and stress fields corresponding to the initial incident wave along the inhomogeneity boundary ($r = R + h$). According to this solution, the shear stress due to the scattering field of the inhomogeneity at the crack site can be obtained by

$$\begin{aligned} \tau_{yz}^f(x) &= \tau_{rz}(\bar{r}, \bar{\phi}) \sin\phi + \tau_{\phi z}(\bar{r}, \bar{\phi}) \cos\phi \\ &= [F_3(x)]\{A\} + \{F_4(x)\} \end{aligned} \tag{39}$$

where $[F_3(x)]$ and $F_4(x)]$ are two known matrices given in Appendix B and

$$\bar{r} = \sqrt{d^2 + (x-a)^2 - 2d(x-a)\cos\theta}, \quad \bar{\phi} = -\frac{\pi}{2} - \tan^{-1}\frac{d\cos\theta - (x-a)}{d\sin\theta} \tag{40}$$

Therefore, the corresponding stress at the collocation points of the crack surface can be expressed as

$$\{\tau^f\} = [S_2]\{A\} + \{S_3\} \tag{41}$$

where

$$[S_2] = \begin{bmatrix} F_3(x_1) \\ F_3(x_2) \\ \cdots \\ F_3(x_N) \end{bmatrix}, \quad \{S_3\} = \begin{bmatrix} F_4(x_1) \\ F_4(x_2) \\ \cdots \\ F_4(x_N) \end{bmatrix} \tag{42}$$

Substituting eqns (41) and (26) into eqn (24) gives

$$\{A\} = [S_1](\{\tau^0\} + [S_2]\{A\} + \{S_3\}) \tag{43}$$

from which $\{A\}$ can be obtained as being

$$\{A\} = (I - [S_1][S_2])^{-1}([S_1]\{\tau^0\} + [S_1]\{S_3\}) \tag{44}$$

The dynamic stress intensity factor at the right tip of the crack in the presence of the inhomogeneity can then be obtained in terms of A_n ($n = 1, 2, \ldots, N$) as being

$$K_{\mathrm{III}} = \mu_M \sqrt{\pi a} \sum_{n=1}^{N} A_n \tag{45}$$

4. Results and discussions

The theoretical analysis described in previous sections is used to investigate the effect of the pertinent parameters upon the dynamic stress intensity factor at the crack under an incident antiplane harmonic wave.

To verify the validity of the current method, consider first the quasistatic antiplane interaction between a perfectly bonded circular inhomogeneity and a collinear semi-infinite crack with an initial stress intensity factor (K_0). The solution of this problem can be predicted by the current method for a relatively large crack length ($a/R > 3$). The normalized stress intensity factor ($K^* = K_{\mathrm{III}}/K_0$) predicted by Turska-Klebek and Sokolowski (1984) is compared with that calculated by the current method in Fig. 3 for different material combinations using twenty terms in the Chebyshev polynomial expansion and forty terms in the Bessel function expansion. In view of the excellent agreement observed between the two, even for the case where the inhomogeneity is very close to the crack tip ($e = d - R = 0.1R$), the number of terms used in this example was retained for the remainder of this study.

Consider now the case of an arbitrarily located inhomogeneity interacting with a crack. The

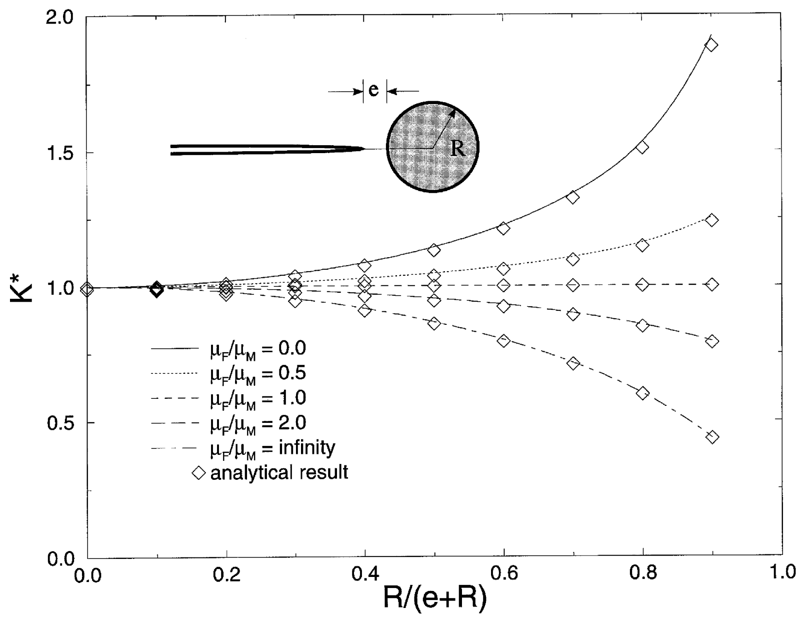

Fig. 3. Quasistatic interaction between a crack and an inhomogeneity.

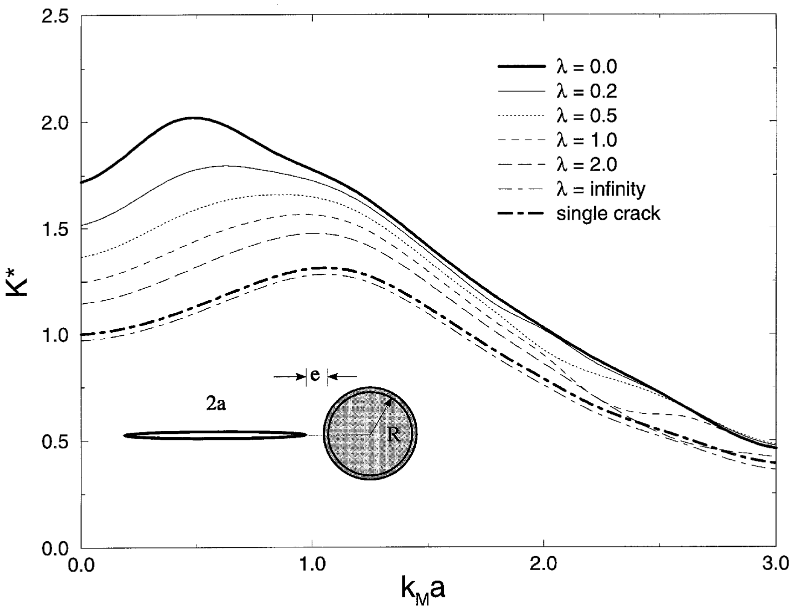

Fig. 4. Effect of the frequency upon the normalized dynamic stress intensity factor K^* for a normal incident wave.

present formulations predict the dependence of the normalized stress intensity factors ($K^* = K_{\mathrm{III}}/\tau\sqrt{\pi a}$) upon the location and size (d/a, θ and R/a) of the inhomogeneity, the material combination, the normalized interface stiffness ($\lambda = a\beta/\mu_M$), the normalized frequency ($k_M a$) and the incident angle (Γ) of the incident wave. It should be recognized that only the amplitude of the complex dynamic stress intensity factor at the right tip of the crack is considered in the following figures.

Figure 4 shows the variation of K^* vs the normalized wave number $k_M a$ for different interface stiffness (λ). A normal incident wave is considered and it is assumed that $e/a = 0.2$, $R = a$, $h/a = 0.04$ and $c_M = c_F$, $\rho_F = \rho_M$ with ρ being the mass density. The well-known overshoot phenomenon for the single crack problem is observed for all the cases examined. The figure indicates that a decrease in the interface stiffness will lower the characteristic frequency at which K^* reaches the maximum value.

Figure 5 shows the variation of K^* of a crack subjected to an oblique incident wave ($\Gamma = 45^\circ$) with $k_M a$ for a collinear case, where $e/a = 0.2$, $R = a$, $h/a = 0.04$ and $c_M = 0.5c_F$, $\rho_M = \rho_F$. In this case, a perfectly bonded inhomogeneity ($\lambda = \infty$) provides a shielding effect at the matrix crack for $k_M a < 1.8$. However, with the decrease in the interface stiffness ($\lambda < 2.0$), such a shielding effect changes to amplification.

Figure 6 shows the variation of K^* vs frequency for the case where the inhomogeneity is directly above the crack. In this case, the normal incident wave ($\Gamma = 90^\circ$) is considered and it is assumed that $d^2 = a^2 + (\delta + R)^2$, $\delta = 0.5a$, $R = a$, $h/a = 0.04$, $c_M = 0.5c_F$ and $\rho_M = \rho_F$. It is interesting to observe that for high frequencies ($k_M a > 1.0$), the inhomogeneity provides a shielding effect at the crack for all the interface conditions considered. For the case where the inhomogeneity is placed directly below the crack with $\Gamma = 90^\circ$, the stress field at the crack tip is reduced (Fig. 7).

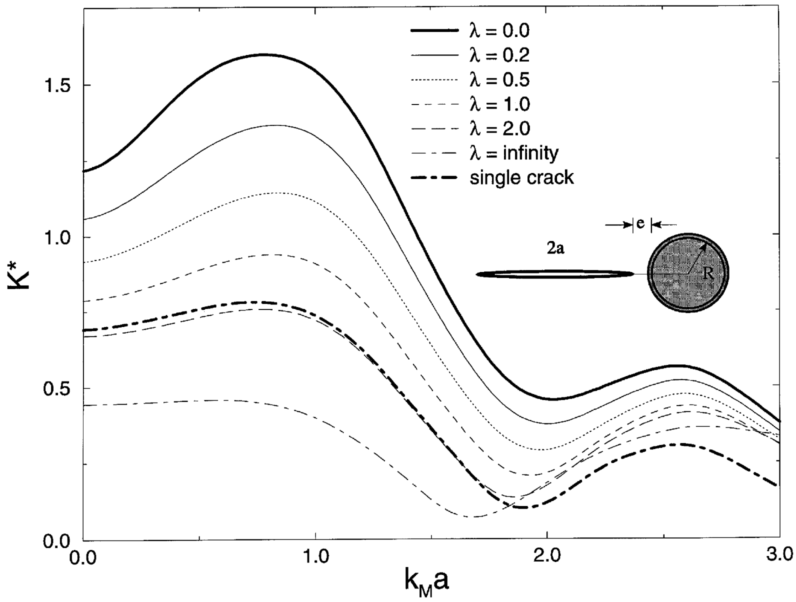

Fig. 5. Effect of the frequency upon the normalized dynamic stress intensity factor K^* for an oblique incident wave.

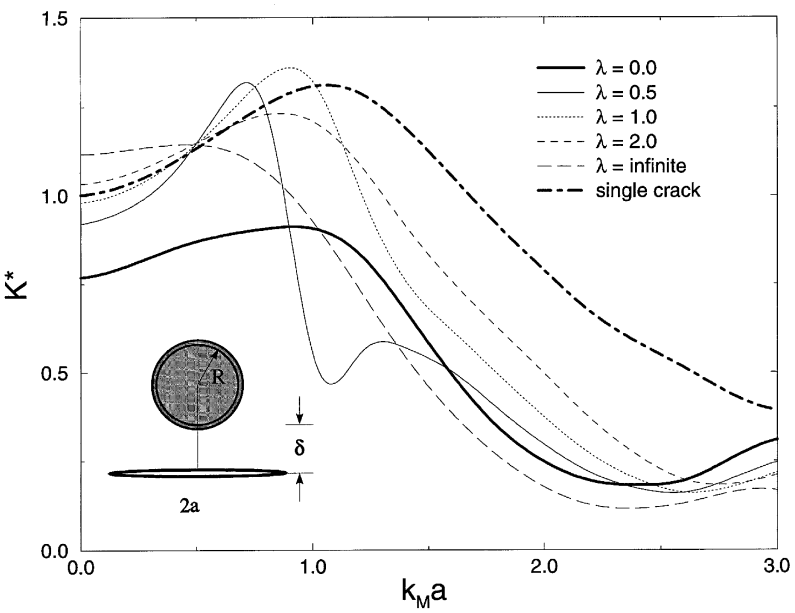

Fig. 6. Effect of an inhomogeneity above the matrix crack upon the normalized dynamic stress intensity factor K^*.

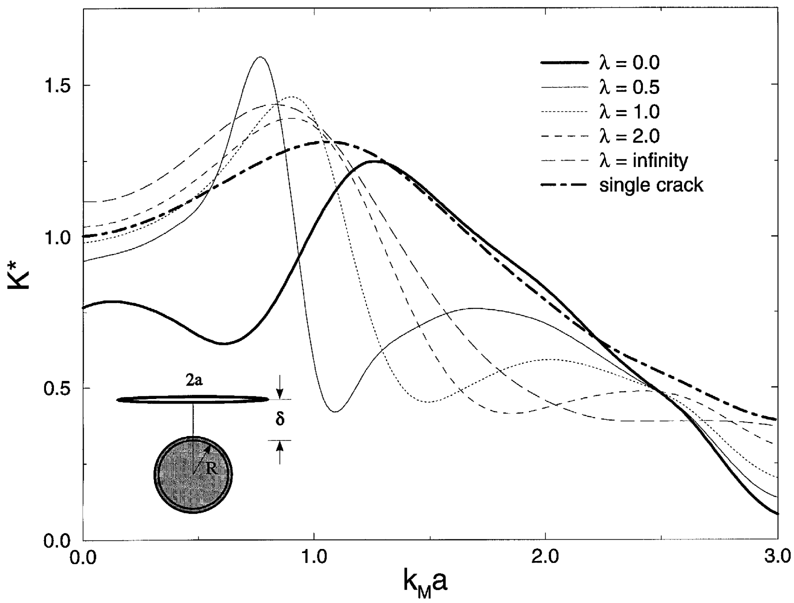

Fig. 7. Effect of an inhomogeneity below the matrix crack upon the normalized dynamic stress intensity factor K^*.

Let us now consider some interesting limiting cases of the examined system. When $c_M/c_F \to 0$ and $\rho_M/\rho_F =$ constant, the inhomogeneity reduces to a rigid inclusion with a finite mass density. In addition, when $\mu_M/\mu_F \to \infty$ and $\rho_M \to \rho_F \to \infty$, the inhomogeneity reduces to a hole which corresponds to a fully debonded inhomogeneity.

5. Concluding remarks

A general analytical solution is provided to the dynamic interaction problem of a matrix crack with an arbitrarily located inhomogeneity with a distinct interphase under antiplane loading. The analysis is based upon the use of a newly developed Pseudo-incident wave method. This method can be generalized to treat more complex interaction problems involving multiple cracks and inhomogeneities.

The validity and versatility of the present solution have been demonstrated by application to specific examples. Furthermore, the effect of the location of the inhomogeneity, the material combination and the frequency of the incident wave upon the dynamic stress intensity factor of the matrix crack and the resulting shielding and amplification effects are examined and discussed.

Acknowledgements

This work was supported in part by the Natural Sciences and Engineering Research Council of Canada (CRD program) and in part by ALCOA Foundation of the U.S.A. The authors also wish to thank one of the reviewers for the constructive comments.

Appendix A

The matrix $[S_1]$ used in eqn (24) is given by

$$[S_1]^{-1} = [s_{lj}], \quad s_{lj} = \frac{\sin\left(\dfrac{jl\pi}{N+1}\right)}{\sin\left(\dfrac{j\pi}{N+1}\right)} + g_j(x_l), \quad j, l = 1, 2, \ldots, N$$

where

$$g_j(x) = \begin{cases} (-1)^n a \displaystyle\int_0^\infty \left(\frac{\alpha}{s} - 1\right) J_j(sc)\cos(sx)\,\mathrm{d}s & j = 2n+1 \\ (-1)^{(n+1)} a \displaystyle\int_0^\infty \left(\frac{\alpha}{s} - 1\right) J_j(sc)\sin(sx)\,\mathrm{d}s & j = 2n \end{cases}$$

$$\alpha = \begin{cases} \sqrt{s^2 - k_M^2} & |s| \geqslant k_M \\ -i\sqrt{k_M^2 - s^2} & |s| < k_M \end{cases}$$

in which J_j is the first kind Bessel function of order j, and

$$x_l = a\cos\left(\frac{l}{N+1}\pi\right), \quad l = 1, 2, \ldots, N$$

being the collocation points along the crack surface.

In addition, the functions p_j and $\bar{p}_j$ in eqns (29) and (30) are given by

$$p_j(x, y) = -a \begin{cases} (-1)^n \displaystyle\int_0^\infty \frac{1}{s} J_j(sc)\cos(sx)\, e^{-\alpha|y|}\,\mathrm{d}s & j = 2n+1 \\ (-1)^{(n+1)} \displaystyle\int_0^\infty \frac{1}{s} J_j(sc)\sin(sx)\, e^{-\alpha|y|}\,\mathrm{d}s & j = 2n \end{cases}$$

$$\overline{p_j}(x, y) = X_j(x, y)\cos\phi + Y_j(x, y)\sin\phi$$

where

$$X_j(x, y) = \mu_M a \begin{cases} (-1)^n \displaystyle\int_0^\infty \frac{\alpha}{s} J_j(sc)\cos(sx)\, e^{-\alpha|y|}\,\mathrm{d}s & j = 2n+1 \\ (-1)^{(n+1)} \displaystyle\int_0^\infty \frac{\alpha}{s} J_j(sc)\sin(sx)\, e^{-\alpha|y|}\,\mathrm{d}s & j = 2n \end{cases}$$

and

$$Y_j(x,y) = \mu_M a\,\mathrm{sgn}(y) \begin{cases} (-1)^n \displaystyle\int_0^\infty J_j(sc)\sin(sx)\,e^{-\alpha|y|}\,\mathrm{d}s & j = 2n+1 \\ (-1)^n \displaystyle\int_0^\infty J_j(sc)\cos(sx)\,e^{-\alpha|y|}\,\mathrm{d}s & j = 2n \end{cases}$$

Appendix B

The matrices in eqn (39) are given by

$$[F_3(x)] = [W_1(\bar{r},\bar{\theta})]\sin\phi + [W_3(\bar{r},\bar{\theta})]\cos\phi, \quad [F_4(x)] = [W_2(\bar{r},\bar{\theta})]\sin\phi + [W_4(\bar{r},\bar{\theta})]\cos\phi$$

where

$$[W_1(r,\theta)] = \mu_M k_M \sum_{n=0}^{\infty} H_n^{(1)\prime}(k_M r)\{1,0\}[H_n] \int_0^{2\pi} \begin{Bmatrix} F_1(\xi) \\ F_2(\xi) \end{Bmatrix} (e^{-in(\xi-\phi)} + e^{in(\xi-\phi)})\,\mathrm{d}\xi$$

$$[W_2(r,\theta)] = \mu_M k_M \sum_{n=0}^{\infty} H_n^{(1)\prime}(k_M r)\{1,0\}[H_n] \int_0^{2\pi} \begin{Bmatrix} w^0(\xi) \\ \tau_{rz}^0(\xi) \end{Bmatrix} (e^{-in(\xi-\phi)} + e^{in(\xi-\phi)})\,\mathrm{d}\xi$$

$$[W_3(r,\theta)] = i\mu_M \sum_{n=0}^{\infty} nH_n^{(1)}(k_M r)\{1,0\}[H_n] \int_0^{2\pi} \begin{Bmatrix} F_1(\xi) \\ F_2(\xi) \end{Bmatrix} (e^{-in(\xi-\phi)} - e^{in(\xi-\phi)})\,\mathrm{d}\xi$$

$$[W_2(r,\theta)] = i\mu_M \sum_{n=0}^{\infty} nH_n^{(1)\prime}(k_M r)\{1,0\}[H_n] \int_0^{2\pi} \begin{Bmatrix} w^0(\xi) \\ \tau_{rz}^0(\xi) \end{Bmatrix} (e^{-in(\xi-\phi)} - e^{in(\xi-\phi)})\,\mathrm{d}\xi$$

References

Achenbach, J.D., 1973. Wave Propagation in Elastic Solids. North-Holland Publishing Company, Amsterdam.

Achenbach, J.D., Zhu, H., 1990. Effect of interphases on micro and macromechanical behavior of hexagonal-array fiber composites. Journal of Applied Mechanics 57, 956–963.

Chen, T., Dvorak, G.J., Benveniste, Y., 1990. Stress fields in composites reinforced by coated cylindrically orthotropic fibres. Mechanics of Materials 9, 17–32.

Chiu, H.-P., Jeng, S.M., Yang, J.-M., 1994. Effect of fiber coating on the tensile and transverse creep behavior of SCS—6/Ti_3Al composite. In: Lin, R.Y., Fishman, S.G. (Eds.), Control of Interfaces in Metal and Ceramics Composites. The Minerals, Metals and Materials Society, Warrendale, Pennsylvania, pp. 119–132.

Datta, S.K., Ledbetter, H.M., Shindo, Y., Shah, A.H., 1988. Phase velocity and attenuation of plane elastic waves in a particle-reinforced composite medium. Wave Motion 10, 171–182.

Drzal, L.T., Madhukar, M., 1993. Fibre–matrix adhesion and its relationship to composite mechanical properties. J. Mater. Sci. 28, 569–610.

Evans, A.G., 1990. Perspective on the development of high-toughness ceramics. J. Am. Ceram. Soc. 73, 187–206.

Hashin, Z., 1990. Thermoelastic properties of fibre composites with imperfect interface. Mech. Mater. 8, 333–348.

Jones, J.P., Whittier, J.S., 1967. Waves at a flexibly bonded interface. Journal of Applied Mechanics December, 905–909.

Karpur, P., Matikas, T.E., Krishnamurthy, S., 1994. Ultrasound as a tool for the development of aerospace structural titanium and ceramic matrix composites. In: Lin, R.Y., Fishman, S.G. (Eds.), Control of Interfaces in Metal and Ceramics Composites. The Minerals, Metals and Materials Society, Warrendale, Pennsylvania, pp. 241–253.

Karpur, P., Matikas, T.E., Krishnamurthy, S., 1995. Ultrasonic characterization of the fibre–matrix interphase/interface for mechanics of continuous fibre reinforced metal matrix and ceramic matric composites. Composites Engineering 5, 697–711.

Matikas, T.E., Karpur, P., 1993. Ultrasonic reflectivity technique for the characterization of fibre–matrix interface in metal matrix composites. Journal of Applied Physics 74, 228–236.

McCullough, R.L., 1971. Concepts of Fiber–Resin Composites. Marcel Dekker, New York.

Meguid, S.A., Wang, X.D., 1994. The dynamic interaction of a microcrack with a main crack under antiplane loading. International Journal of Solids and Structures 31, 1085–1097.

Mura, T., 1987. Micromechanics of Defects in Solids, 2nd ed. Martinus Nijhoff, Boston.

Olssen, P., Datta, S.K., Bostrom, A., 1990. Elastodynamic scattering from inclusions surrounded by thin interface layers. Journal of Applied Mechanics 57, 672–676.

Ramesh, K.T., Ravichandran, G., 1990. Dynamic behavior of a boron carbide–aluminum cermet: experiments and observations. Mechanics of Materials 10, 19–29.

Schafbuch, P., Thompson, R.B., Rizzo, F., 1990. Application of boundary element method to elastic wave scattering by irregular defects. J. Nondest. Eval. 9, 113–127.

Turska-Kebek, E., Sokolowski, M., 1984. On the influence of defects upon stress concentration at the crack. Arch. Mech. 36, 121–126.

Veazie, D.R., Qu, J., 1995. Effects of interphases on the transverse stress–strain behaviour in unidirectional fibre reinforced metal matrix composites. Composites Engineering 5, 597–610.

Wang, Q., Chiang, F.-P., 1996. Experimental characterization of interphase mechanical properties of composites. Composites 27B, 123–128.

Wass, A.M., 1992. Effect of interphases on compressive strength of unidirectional composites. Journal of Applied Mechanics 59, s183–s188.